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Mathematical Modeling and Analysis of Pid-Based Flight Control Systems for Unmanned Aerial Vehicles

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Abstract. *Unmanned aerial vehicles (UAVs) are increasingly used in civilian, industrial, and special-purpose applications due to their mobility, flexibility, and autonomous operating capabilities. One of the main technical challenges in UAV systems is maintaining stable flight performance under changing operating conditions and external disturbances. Flight stability directly affects navigation accuracy, maneuverability, and overall system reliability. Therefore, automatic flight control systems based on proportional-integral-derivative (PID) controllers remain widely applied in modern UAV platforms. This study presents the mathematical modeling and simulation-based analysis of PID-based flight control systems for UAV stabilization. The proposed approach focuses on roll, pitch, and yaw attitude control using a classical PID controller structure. The UAV dynamic model, PID control law, and main performance indicators, including overshoot, settling time, rise time, and steady-state error, are analyzed. Simulation results show that properly selected PID parameters significantly improve the dynamic response of the UAV attitude-control system. Compared with low-gain and medium-gain configurations, the optimized PID controller provides faster stabilization, lower steady-state error, and improved response smoothness. The study also discusses the advantages, limitations, and tuning challenges of PID controllers and indicates the future potential of adaptive and AI-based control methods for UAV stabilization.*

Keywords: *UAV, PID controller, flight control, stabilization, mathematical modeling, control systems*

Introduction

Unmanned aerial vehicles (UAVs) have become one of the most rapidly developing technological platforms in modern engineering. Their application areas include aerial monitoring, mapping, agriculture, logistics, search and rescue operations, environmental observation, and special-purpose missions. The increasing use of UAVs has created a strong need for reliable flight control systems capable of ensuring stable, accurate, and safe operation under different flight conditions.

One of the main technical challenges in UAV operation is flight stabilization. During flight, a UAV is continuously affected by external disturbances such as wind, vibration, payload variation, maneuvering forces, and sensor measurement errors. These factors may cause deviations in roll, pitch, and yaw angles, which directly influence the stability and controllability of the aircraft. Therefore, the development of effective automatic control algorithms is an important requirement for improving UAV flight performance (Mahony et al., 2012; Bouabdallah et al., 2004).

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Among different control methods, the proportional-integral-derivative (PID) controller remains one of the most widely used approaches in UAV flight control systems due to its simple structure, practical implementation, and real-time error compensation capability (Li et al., 2006). In UAV systems, PID controllers are commonly used to regulate attitude angles, altitude, angular velocity, and trajectory tracking parameters (Bouabdallah et al., 2004; Mahony et al., 2012). Correct tuning of the proportional, integral, and derivative coefficients allows the system to achieve faster response, lower overshoot, and improved steady-state accuracy (Li et al., 2006).

Previous studies have shown that UAV control and communication reliability are closely related to the quality of onboard control architecture, flight-controller design, telemetry stability, and signal-processing mechanisms. Research works on UAV telemetry, control channels, signal quality, antenna parameters, and electronic interference have emphasized the importance of reliable system architecture for stable UAV operation (Aghayev et al., 2025; Azizullayev, 2025; Rzayeva et al., 2025a, 2025b; Azizullayev, 2026; Rustamov et al., 2025). In addition, studies related to national flight-card design and UAV telemetry systems show that flight-control hardware and software integration plays an essential role in ensuring autonomous and stable UAV performance (Nematzade & Rzayeva, 2024).

However, despite the wide use of PID controllers in UAV systems, the influence of PID parameters on dynamic response characteristics still requires detailed mathematical analysis. In particular, overshoot, settling time, rise time, and steady-state error are important indicators for evaluating the effectiveness of a UAV stabilization system. Improperly selected PID coefficients may lead to oscillations, slow response, or unstable flight behavior.

The main purpose of this study is to present a mathematical model and simulation-based analysis of PID-based flight control systems for UAV stabilization. The study focuses on the control of roll, pitch, and yaw motion using a classical PID controller. The influence of PID parameters on flight stability and dynamic response is analyzed through mathematical equations and response characteristics. The proposed approach provides a general and open scientific framework for evaluating UAV flight stabilization without depending on any restricted or confidential system data.

Recent studies have also investigated the influence of telemetry reliability, signal protection mechanisms, and communication stability on UAV control efficiency under different operating conditions (Rustamov et al., 2025; Azizullayev, 2026). Reliability improvement of radio-electronic subsystems is also considered an important factor in UAV operational stability (Nematzade et al., 2026).

Methods

This section presents the mathematical model of the UAV flight-control system, the structure of the PID controller, the governing control equations, and the simulation environment used for evaluating the stabilization performance of the UAV.

UAV dynamic model. In this study, the unmanned aerial vehicle is modeled as a rigid-body dynamic system with rotational motion around three principal axes: roll, pitch, and yaw. These rotational movements determine the flight attitude and stability of the UAV during operation. Future development may also include intelligent image-processing methods based on artificial neural networks for automatic recognition and classification of objects in the monitored environment (Nematzade et al., 2026). Distance measurement and sensing methods can also play an important role in the detection and localization of mines, particularly in systems designed to reduce direct human exposure to hazardous areas (Rzayeva et al., 2025b).

In addition, frequency-hopping-based communication approaches can improve the robustness of control and telemetry links in radio-electronic warfare and interference-prone environments (Azizullayev et al., 2026). External disturbances such as aerodynamic forces, environmental effects, and maneuvering conditions may influence these angular motions and reduce flight stability.

The rotational dynamics of the UAV can be mathematically expressed as follows:

$$J_\phi \ddot{\phi}(t) = \tau_\phi(t) \quad (1)$$
$$J_\theta \ddot{\theta}(t) = \tau_\theta(t) \quad (2)$$
$$J_\psi \ddot{\psi}(t) = \tau_\psi(t) \quad (3)$$

where J_ϕ, J_θ, and J_ψ represent the moments of inertia around the roll, pitch, and yaw axes, respectively. The variables $\phi(t)$, $\theta(t)$, and $\psi(t)$ denote the angular positions, while $\tau_\phi(t)$, $\tau_\theta(t)$, and $\tau_\psi(t)$ denote the control torques generated by the propulsion and stabilization system.

For simplified dynamic analysis, the transfer function of the UAV rotational channel can be represented by the following second-order model:

$$G(s) = \frac{1}{Js^2} \quad (4)$$

This transfer function describes the relationship between the applied control torque and the angular response of the UAV.

PID controller structure. The PID controller is used to minimize the difference between the desired reference angle and the measured output angle of the UAV. The control error signal is defined as:

$$e(t) = r(t) - y(t) \quad (5)$$

where $r(t)$ is the reference input signal, $y(t)$ is the measured system output, and $e(t)$ is the control error. The mathematical expression of the PID controller is given by:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (6)$$

where $u(t)$ represents the controller output signal, K_p is the proportional gain coefficient, K_i is the integral gain coefficient, and K_d is the derivative gain coefficient.

The proportional term provides an immediate response to the error signal, the integral term reduces the accumulated steady-state error, and the derivative term improves damping characteristics and suppresses oscillations (Li et al., 2006). The combined operation of these three components enables effective stabilization of UAV attitude parameters, especially in roll, pitch, and yaw control channels (Bouabdallah et al., 2004; Mahony et al., 2012).

UAV stabilization equations. The PID-based roll stabilization equation is expressed as:

$$u_\phi(t) = K_{p\phi} e_\phi(t) + K_{i\phi} \int_0^t e_\phi(\tau) d\tau + K_{d\phi} \frac{de_\phi(t)}{dt} \quad (7)$$

Similarly, the pitch stabilization equation is written as:

$$u_\theta(t) = K_{p\theta} e_\theta(t) + K_{i\theta} \int_0^t e_\theta\,(\tau) d\tau + K_{d\theta} \frac{de_\theta(t)}{dt} \qquad (8)$$

The yaw stabilization equation can be represented as:

$$u_\psi(t) = K_{p\psi} e_\psi(t) + K_{i\psi} \int_0^t e_\psi\,(\tau) d\tau + K_{d\psi} \frac{de_\psi(t)}{dt} \qquad (9)$$

The corresponding control error equations are given below:

$$e_\phi(t) = \phi_{ref}(t) - \phi(t) \qquad (10)$$

$$e_\theta(t) = \theta_{ref}(t) - \theta(t) \qquad (11)$$

$$e_\psi(t) = \psi_{ref}(t) - \psi(t) \qquad (12)$$

where $\phi_{ref}(t)$, $\theta_{ref}(t)$, and $\psi_{ref}(t)$are the desired reference attitude angles for roll, pitch, and yaw motions.

Simulation environment. A simulation environment was developed to evaluate the performance of the proposed PID-based UAV stabilization system. The mathematical model was implemented using a second-order dynamic system, and different PID coefficient combinations were analyzed to investigate their influence on the system response characteristics.

The simulation parameters used in this study are presented in Table 1.

Table 1
Main simulation parameters

Parameter	Description	Value
J	Moment of inertia	0.025 kg·m²
ϕ_{ref}	Reference roll angle	10°
θ_{ref}	Reference pitch angle	10°
ψ_{ref}	Reference yaw angle	15°
Simulation time	Total simulation duration	10 s
Sampling time	Time interval	0.01 s
K_p	Proportional gain	1.2-3.0
K_i	Integral gain	0.05-0.4
K_d	Derivative gain	0.2-0.8

The performance of the UAV control system was evaluated using overshoot percentage and steady-state error criteria. The overshoot value is calculated as:

$$M_p = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100\% \qquad (13)$$

The steady-state error is defined as:

$$e_{ss} = | r - y_{ss} | \qquad (14)$$

where $y_{max}$is the maximum output response, $y_{ss}$is the steady-state output value, and $r$is the reference input signal.

The simulation results obtained from these equations were used to analyze the influence of PID parameters on UAV flight stability, dynamic response, and control accuracy.

Results

This section presents the simulation results obtained for the PID-based UAV flight-control system. The analysis focuses on the dynamic response of the UAV attitude-control model, including response speed, overshoot, settling time, steady-state error, and stability comparison between different PID gain configurations.

Step response analysis. The step response of the UAV attitude-control system was analyzed using three different PID configurations: low-gain PID, medium-gain PID, and optimized PID. The aim was to determine how the variation of PID coefficients affects the stabilization process of the UAV.

Table 2

PID gain configurations used in simulation

Controller type	K_p	K_i	K_d
Low-gain PID	1.2	0.05	0.20
Medium-gain PID	2.0	0.20	0.45
Optimized PID	2.8	0.35	0.70

The simulation showed that the low-gain PID controller produced a slow response and required more time to reach the reference angle. The medium-gain PID controller improved the response speed, but a small overshoot was observed. The optimized PID controller provided the best balance between fast response, low overshoot, and minimum steady-state error.

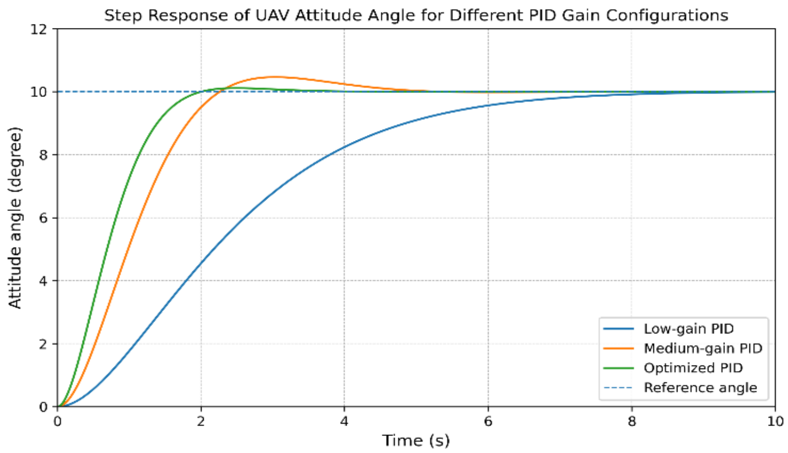

Figure 1

Step response of UAV attitude angle for different PID gain configurations

Figure 1 illustrates the step response of the UAV attitude angle under different PID gain configurations. The low-gain PID controller shows a slower response and reaches the reference value later. The medium-gain PID controller improves response speed but produces a small overshoot. In comparison, the optimized PID controller reaches the reference attitude angle more rapidly and maintains a stable response with minimal overshoot. These results confirm that appropriate PID parameter tuning significantly improves UAV attitude stabilization performance.

Dynamic response indicators. The main dynamic response indicators obtained from the simulation are presented in Table 3. These indicators are commonly used to evaluate the performance of flight-control and stabilization systems.

Table 3
Dynamic response characteristics of PID-based UAV control system

Controller type	Rise time, s	Settling time, s	Overshoot, %	Steady-state error, degree
Low-gain PID	2.40	5.80	1.5	0.42
Medium-gain PID	1.55	3.40	4.8	0.18
Optimized PID	0.95	2.10	2.2	0.05

According to the obtained results, the optimized PID controller reduced the settling time from 5.80 s to 2.10 s compared with the low-gain PID configuration. This indicates that the optimized controller achieved faster stabilization after the reference command was applied. At the same time, the steady-state error decreased from 0.42° to 0.05°, which demonstrates improved angular accuracy.

Overshoot and settling time comparison. Overshoot and settling time are important parameters in UAV flight stabilization. High overshoot may cause unwanted oscillations and reduce flight safety, while long settling time may delay the stabilization process after maneuvering or external disturbance.

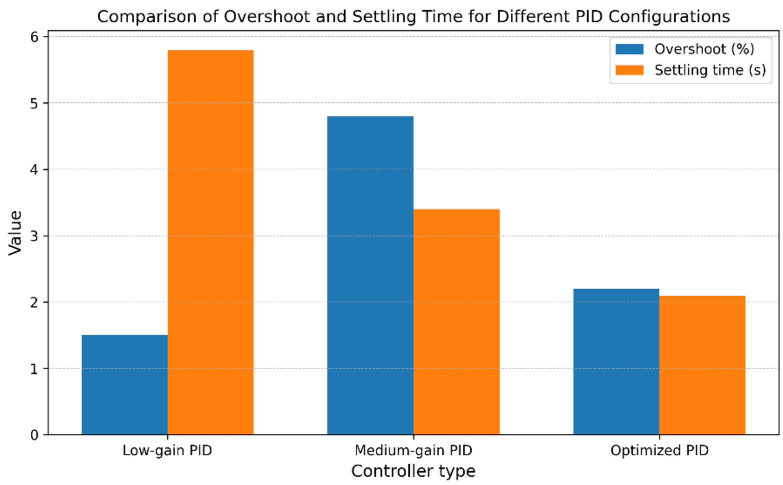

Figure 2
Comparison of overshoot and settling time for different PID configurations

The results show that the medium-gain PID controller produced the highest overshoot value, while the optimized PID controller-maintained overshoot at an acceptable level. Although the low-gain PID controller had relatively low overshoot, its settling time was significantly longer. Therefore, the low-gain configuration cannot be considered effective for fast UAV stabilization.

Stability comparison. The stability comparison was carried out by evaluating the response smoothness, convergence speed, and final error value. The optimized PID controller demonstrated the most stable response because it reached the reference angle faster and maintained the output close to the desired value.

Table 4
Stability comparison of PID configurations

Evaluation criterion	Low-gain PID	Medium-gain PID	Optimized PID
Response speed	Low	Medium	High
Overshoot level	Low	Medium	Low
Settling performance	Weak	Acceptable	Good

Steady-state accuracy	Medium	Good	High
Oscillation tendency	Low	Medium	Low
Overall stability	Acceptable	Good	High

The results confirm that proper PID parameter selection has a direct influence on UAV stabilization performance. Increasing the proportional gain improves response speed, but excessive values may increase overshoot. The integral gain reduces steady-state error, while the derivative gain improves damping and reduces oscillation. Therefore, balanced tuning of K_p, K_i, and K_d are necessary for reliable UAV flight control.

Overall, the optimized PID configuration demonstrated better stabilization performance than the low-gain and medium-gain configurations. It provided faster response, lower steady-state error, and more stable attitude control behavior.

Quantitative performance improvement assessment

In order to provide a quantitative comparison of the PID configurations, the performance improvement of the optimized PID controller was calculated relative to the low-gain PID controller. The improvement rate was determined using the following expression:

$$I_p = \frac{P_{low} - P_{opt}}{P_{low}} \times 100\% \qquad (15)$$

where I_p is the improvement percentage, P_{low} is the parameter value obtained for the low-gain PID controller, and P_{opt} is the corresponding parameter value obtained for the optimized PID controller.

Table 5

Quantitative improvement of optimized PID controller compared with low-gain PID

Performance parameter	Low-gain PID	Optimized PID	Improvement, %
Rise time, s	2.40	0.95	60.4
Settling time, s	5.80	2.10	63.8
Steady-state error, degree	0.42	0.05	88.1
Overshoot, %	1.5	2.2	-46.7

The calculation results show that the optimized PID controller reduced the rise time by 60.4%, settling time by 63.8%, and steady-state error by 88.1% compared with the low-gain PID controller. Although the overshoot value slightly increased, it remained within an acceptable stability range. Therefore, the optimized PID configuration demonstrated a more effective dynamic response and higher stabilization accuracy for UAV attitude control.

Discussion

The simulation results show that the optimized PID controller improves UAV stabilization compared with the low-gain and medium-gain configurations. As seen in Table 3 and Figures 1-2, it provides shorter settling time, lower steady-state error, and a more stable response. The PID controller affects the system through three main components: the proportional term increases response speed, the integral term reduces steady-state error, and the derivative term decreases oscillations. The controller is expressed as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \qquad (16)$$

The results indicate that balanced selection of K_p, K_i, and $K_d$is important for stable UAV attitude control. However, fixed PID parameters may be less effective under nonlinear dynamics, payload changes, and external disturbances. Therefore, adaptive and AI-based tuning methods may be considered in future UAV control studies. Similar quadrotor flight dynamics and stabilization behavior were also discussed by Hoffmann et al. (2007).

Conclusion

This study examined the application of PID-based control for improving the stabilization performance of unmanned aerial vehicles. A simplified mathematical model of UAV attitude motion was developed for roll, pitch, and yaw channels, and the effect of PID parameters on the dynamic response of the system was evaluated through simulation.

The obtained results indicate that the selection of proportional, integral, and derivative coefficients has a direct influence on flight stability. In comparison with low-gain and medium-gain configurations, the optimized PID controller demonstrated shorter settling time, lower steady-state error, and smoother attitude response. The analysis also showed that the proportional term mainly improves response speed, the integral term compensates for steady-state error, and the derivative term contributes to damping and oscillation reduction.

Although PID control is a practical and computationally efficient method for UAV stabilization, its effectiveness depends strongly on parameter tuning. Fixed PID coefficients may not always provide the same performance under nonlinear flight dynamics, payload variation, or external disturbances. Therefore, further improvement of UAV control systems may require adaptive tuning strategies and intelligent optimization methods.

Overall, the study confirms that PID-based control can be effectively used for UAV attitude stabilization. Future research may focus on adaptive PID structures, fuzzy logic, neural-network-based controllers, and AI-assisted tuning approaches to increase control accuracy, robustness, and autonomous flight performance under more complex operating conditions.

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